ELECTROMAGNETIC PROPERTIES OF OFF-SHELL PARTICLES AND GAUGE INVARIANCE

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Abstract

Electromagnetic properties of off-shell particles are discussed on the sis of a purely electromagnetic reaction: virtual Compton scattering off a proton. It is shown that the definition of off-shell electromagnetic form factors is not gauge invariant and they cannot be investigated in practice. Using only fundamental requirements of gauge invariance it is demonstrated that off-shell effects are cancelled in the longitudinal components of the total conserved current by the "minimal" contact current, while off-$shellness$ appears only in the transverse (gauge invariant) non-pole part. This provides the possibility to introduce an *on-shell extrapolated* form factor F_1^+ in a gauge invariant way for the time-like region $4m_e^2 < q^2 < 4M^2$. The form factors $F_{1,2}^-$ are removed from the total conserved current and do not affect observables.

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I. INTRODUCTION

Electromagnetic (EM) properties of off-shell (virtual) particles have been the subject of intensive investigations[1−4]. It was shown that the number of off-shell form factors (FF) in

the general $\gamma^\star NN$ vertex with one (two)- off-shell nucleon(s) increases[2] to 4 (6) compared to two FF for the free nucleon. Dispersion relations[1] for off-shell FF were introduced by Bincer. Various one-loop calculations[2−4] suggested their model-dependence and possible influence the observables in the electron-nucleus scattering.

It is well known that any off-mass-shell extension of a matrix element is not unique: various representations of the field operators give rise to different off-shell Green functions and off-shell matrix elements, while all of these lead to the same S-matrix. Off-shell properties of Green functions and in particular their dependence on the representation of the Lagrangian have been discussed in the framework of field theory for a long time[5]. Recently the method of effective Lagrangians was applied to real Compton scattering where the representation dependence of off-shellness has been pointed out[6]. The possibility to move off-shell effects from irreducible vertices of the Born current to the regular part of the amplitude was used[6] to obtain a unique definition for polarizabilities in Compton scattering on nucleons. Also in chiral perturbation theory applied to real Compton scattering off the pion, it was shown[7] that off-shell effects depend not only upon the model of Lagrangian, but also on its representation. The possibility to shift any explicit off-shell dependence of the irreducible three-point Green function to the regular part of the amplitude has also been used to derive[8,9] the low energy theorem for the virtual Compton scattering (VCS).

From the general point of view it is clear that the definition of an off-*shell* FF is directly connected with the possibility to introduce a *one-body* off-*shell* EM current independently of the full EM reaction amplitude. However, due to requirements of gauge invariance one- and many-body currents cannot be considered separately[10]. Since the one-body off-shell current is not conserved itself, the definition of off-shell FF will not be gauge invariant. Therefore, off-shell effects in the $\gamma^\star NN$ vertex should be considered in connection with the representation of the full conserved current.

In the space-like region (and also for $q^2 > 4M^2$), where the on-shell $\gamma^\star NN$ vertex corresponds to a physical process, an *on-shell* EM FF can be defined in a gauge invariant manner (they are known, or at least, may be measured), while any off-shell effects may be considered

as corrections and formally transferred to the non-Born part of the amplitude[8,9,11]. On other hand, in the so-called "unphysical" region ($4m_e^2 < q^2 < 4M^2$) for any EM process one of the hadrons in the EM vertices is *off-mass-shell* and hence only *off-shell* EM FF exist. However, their definition cannot be gauge invariant and such a FF cannot be investigated in practice. That is why in the case of a virtual time-like photon it is important to understand whether a representation of the amplitude exists such that the off-shell effects enter only in the non-pole transverse (gauge invariant) components of the total conserved current, whereas the pole-type piece contains only "on-shell extrapolated" FF.

In general there are various ways (which lead to the same physical result) to construct a covariant and gauge invariant VCS amplitude[7–9]. Here we will use a field-theoretical approach[10] formulated in terms of the n-point Green functions, and explore the gauge nature of the EM field (associated with the real photon). The Ward-Takahashi identities (WTI), which connect the (n+1)-point EM Green functions (with EM field) and the hadron n-point Green functions (without EM field), are an important ingredient of the gauge invariant theory in this case.

In this paper we focus on the off-shell effects associated with *virtual* photons in the *reducible* $\gamma^\star NN$ vertices which, in the case of Dirac or scalar particles, do not contain any off-shellness at the "photon point" due to gauge constraint[2,10]. For our goals we need not only to introduce an exactly conserved EM current, but it is also important to define its *irregular* and *regular* pieces (pole and contact current) consistently, from the "same principle", but without special dynamical model for off-shellness. The "minimal insertion" of the gauge field into n-point Green functions provides this on the basis of the fundamental requirements only, and it allows to see how the off-shell effects may be transferred between one- and many-body reaction mechanisms inside the total gauge invariant amplitude in a model independent way. In the "minimal" scheme the off-shell Born current (irregular part) and contact current (regular part) will be defined from the same off-shell $\gamma^\star NN$ vertex for which only the general structure will be assumed. Of course, such a consideration is restricted to only the Dirac part of the γNN vertex associated with *real photon*, since

its “abnormal” (Pauli) part cannot be included in the “minimal scheme” directly, but in principle it may be considered like a “vertex correction” which could be calculated through loop-diagrams. However, in this case a dynamical model for the higher-order hadron Green functions (strong vertices) is essential. As for the $\gamma^\star NN$ vertices, associated with *virtual photons*, in general, both Dirac and Pauli parts may formally be included, since we do not suppose “minimal” procedure in this case and we can use the most general phenomenological expression satisfying fundamental requirements. However, for simplicity we will restrict ourselves to only the Dirac FF in the $\gamma^\star NN$ vertex also, and mention additional effects associated with the Pauli FF if necessary.

We note that the present study differs from previous studies of the VCS[8−9,11] (for completeness see also the refs. in [9]) were based upon the introduction of an “on-shell Born current”, transferring all possible off-shell effects to the unknown regular part; the latter was then parametrised in terms of the invariant functions - “generalized polarizabilities” - on the basis of the fundamental requirements of covariance, CPT and gauge invariance.

The paper is arranged in following way. In Section II a purely EM process $\gamma + p \Longleftrightarrow \gamma^\star + p$ (space- or time-like VCS off a proton) is considered. First, we give the definitions of the various hadrons and EM Green functions (without and with EM field), as well as corresponding WTI to which the last ones should satisfy. Then, using minimal substitution of the gauge field into the corresponding Green functions, the structure of the off-*shell* Born current, the contact current and the total conserved VSC current is obtained on the basis of the same principle, in terms of the *reducible* vertices and $free$ Feynman propagators. We show that off-*shell* form factors cannot be defined in a gauge invariant way, even if $\gamma^\star NN$ vertex satisfy corresponding WTI, and, as a result, they cannot be investigated in reality.

In Section III we deal with the ”off-shellness” in the *regular* and *irregular* pieces of the conserved current. In a model independent way we show that off-shell effects, which appear only for the *virtual* photons in the *reducible* $\gamma^\star NN$ vertices, may be not only moved from the pole-part of the amplitude to its regular part, but also *cancelled* in the charge operators of the total conserved current, if the pole and contact amplitudes are considered consistently.

Additionally, we will see that form factor $F^{-}_{1,2}$, connected with negative energy states of the virtual proton, will be cancelled in the total conserved current and will not influence the observables.

Section IV is devoted to an alternative construction of the total conserved current in terms of the *irreducible* vertices and *full* renormalized propagators, again on the basis of the minimal substitution procedure. Another representation of the "minimal" conserved current for VCS is obtained there. The relation between two different representations of the conserved current, based on the *reducible* and *irreducible* $\gamma^{\star}NN$ vertices, is established. In the framework of the "minimal" scheme, using general properties of the mass-operator, it was shown that the "corrections" to the Born and contact currents, stipulated by the self-energy part, have the same absolute value but opposite signs. So, they cancel one another in the total conserved current in a such a model independent way that the amplitudes defined in terms of the *reducible* vertices with *free* Feynman propagators and *irreducible* ones together with *full* renormalized propagators become the same. At last, for the *real* photons the off-shellness in the *irreducible* vertices may be isolated incide the "correction terms" only and, due to their cancellation in the total current, may be not considered at all. This simply corresponds to the transition to the representation of the conserved current in terms of the *reducible* vertices which are free from the off-shellness due to the gauge constraint.

Section V is devoted to an application of the gauge invariant definition of the "on-shell extended" form factors in the time-like *unphysical* region, which is only possible due to special representation of the total amplitude, and is based on the cancellation of the off-shellnes in the conserved current. The sensitivity of the $e^{+}e^{-}$- asymmetry for the di-lepton production off the proton in a "special" kinematics to such a "gauge invariant" form factors is considered. In Section VI we discuss the results and give the conclusions. Some details and intermediate transformations are given in the Appendices.

II. STRUCTURE OF THE VCS AMPLITUDE

In this section the structure of the gauge invariant nucleon current for VCS off a proton with accounting for the off-shell effects in the EM vertices will be considered. For a Dirac proton (in relation to the *real photon*) or scalar particles a consistent definition of the regular and irregular pieces of the total conserved current, based on the gauge nature of EM field, is possible without a special model for the off-shell behaviour of the FF. To derive this, let us introduce first strong and EM Green's functions, as well as the corresponding WTI to which the lastes should satisfy.

A. Preliminaries

The two-point Green function of the proton, as well as the electromagnetic three- and four-point Green functions are defined as

$$S(x,y) = -i < 0|T\{\psi(x)\bar{\psi}(y)\}|0 >,$$

$$G_{\mu}(x,y,z) =< 0|T\{\psi(x)\bar{\psi}(y)j_{\mu}(z)\}|0 >,$$

$$G_{\mu\nu}(x,y,z,r) =< 0|T\{\psi(x)\bar{\psi}(y)j_{\mu}(z)j_{\nu}(r)\}|0 >,$$

where $j_{\mu}(r) = -\nabla^2 A_{\mu}(r)$ is the EM current operator, $\psi(x)$ denotes an interpolating field operator of the proton, and A_{μ} is EM field satisfying Lorentz condition $\nabla_{\mu}A_{\mu}(r) = 0$; T denotes the time-ordered product.

Using translation invariance the corresponding momentum-space Green functions are

$$(2\pi)^4\delta^4(p-p')S(p) = \int d^4xd^4ye^{i(px-p'y)}S(x,y),$$

$$(2\pi)^4\delta^4(p-p'-q')G_{\mu}(p,p') = \int d^4xd^4yd^4re^{i(px-p'y-q'r)}G_{\mu}(x,y,r),$$

$$(2\pi)^4\delta^4(p+q-p'-q')G_{\mu}(P,q,q') = \int d^4xd^4yd^4re^{i(px+qz-p'y-q'r)}G_{\mu}(x,y,z,r),$$

where $P = p + p'$, and q, q' are the photon momenta assosiated with currents j_{μ} and j_{ν}.

The truncated three- and four-point electromagnetic Green functions, or γNN and $\gamma NN\gamma$ vertices are obtained by "amputation" of the external proton lines:

$$\left\{ \begin{array}{c} \Gamma_\mu(p,p') \\ \Gamma_{\mu\nu}(P,q,q') \end{array} \right\} \quad = \quad S^{-1}(p) \left\{ \begin{array}{c} G_\mu(p,p') \\ G_{\mu\nu}(P,q,q') \end{array} \right\} S^{-1}(p')$$

To be more concrete from now on we will distinguish the half-off-shell *reducible* γNN vertex ($\Gamma_\mu(p,p')$) and the *irreducible* one ($\Gamma^{\rm ir}_\mu(p,p')$)

$$S(p)\Gamma^{\rm ir}_\mu(p,p')S(p') = S_0(p)\Gamma_\mu(p,p')S_0(p'), \tag{1}$$

where $S(p)$ and $S_0(p)$ is the full renormalized and free Feynman propagators.

In the case of $half$-off-$shell$ γNN ir-reducible or reducible vertices (with real or virtual photons) the corresponding WTI have the same form (see Appendix A):

$$q_\mu\Gamma^{\rm ir}_\mu(p,p') = e\{S^{-1}(p) - S^{-1}(p')\}, \tag{2}$$

$$q_\mu\Gamma_\mu(p,p') = e\{S_0^{-1}(p) - S_0^{-1}(p')\}, \tag{3}$$

B. Conserved current in terms of reducible vertices

A conserved current for a purely EM reaction $\gamma + p \Longleftrightarrow \gamma^\star + p$ on a Dirac proton can be obtained using minimal substitution of the EM field into the three-point Green function[10], corresponding to the $\gamma^\star NN$ vertex (with virtual photon). In general this procedure may be carried out in two different ways, namely on either the right-hand or the left-hand side of eq.(1). In the first case the theory will be formulated in terms of the *reducible* vertices and free Feynman propagators, while the second case will lead to the formulation in terms of *irreducible* vertices and full renormalized propagators. Even from the beginning it is evident that the same results for physical observables should be obtained, since a gauge (massless) field is "inserted" into the different sides of identity. In this section we start with the right-hand side of eq.(1) and therefore we concentrate on the off-shellness in the

reducible vertices. (The latter occurs only for virtual photons and does not exist at the "photon point".)

The minimal insertion of the second (real) photon into the external lines of the right-hand side of eq.(1) generates[10] the one-body *off-shell* Born current (see Appendix B)

$$J_{\mu\nu}^{B-off} = e\{\Gamma_\mu(p, p'+q')S_0(p'+q')\gamma_\nu + \gamma_\nu S_0(p-q')\Gamma_\mu(p-q', p')\}. \tag{4}$$

Here $p'(p)$ is initial (final) momentum of the nucleon, $q(q')$ is the virtual (real) photon momentum: $p+q=p'+q'$; e is a charge, and γ_ν is a 4×4 Dirac matrix.

The minimal photon insertion inside the $\gamma^\star NN$ vertex gives rise to the contact ("many-body") current[10], containing only one-body irreducible graphs (diagrams which cannot be disconnected by cutting only any single-particle internal lines). Note, that the requirement of gauge invariance itself does not fix the contact current completely, since it still depends on the "trajectory"[10]. The additional assumption of the *minimal* insertion of the EM field, corresponding to the "minimal (linear) trajectory", makes its definition unique[10]

$$J_{\mu\nu}^{C} = -e\int_0^1 \frac{d\lambda}{\lambda}\frac{d}{dq'_\nu}\{\Gamma_\mu(p, p'+\lambda q') - \Gamma_\mu(p-\lambda q', p')\}. \tag{5}$$

It is easy to check that the derivative of $J_{\mu\nu}^{C}$ cancels corresponding derivative of $J_{\mu\nu}^{B-off}$ hence the total nucleon current

$$J_{\mu\nu}^{N} = J_{\mu\nu}^{B-off} + J_{\mu\nu}^{C} \tag{6}$$

is conserved for any off-shell $\gamma^\star NN$ vertex, independent of its explicit form (initial and final spinors are implied here)

$$q_\mu J_{\mu\nu}^{N} = J_{\mu\nu}^{N} q'_\nu = q_\mu(J_{\mu\nu}^{B-off} + J_{\mu\nu}^{C}) = (J_{\mu\nu}^{B-off} + J_{\mu\nu}^{C})q'_\nu = 0.$$

We note that $J_{\mu\nu}^{B-off}$ and $J_{\mu\nu}^{C}$ are not gauge invariant separately

$$J_{\mu\nu}^{B-off} q'_\nu = -J_{\mu\nu}^{C} q'_\nu = e\{\Gamma_\mu(p, p'+q') - \Gamma_\mu(p-q', p')\} \neq 0,$$

although $q_\mu J_{\mu\nu}^{B-off} = q_\mu J_{\mu\nu}^{C} = 0$. Only the *on-shell* Born current, $J_{\mu\nu}^{B-on}$, is conserved[8], since the corresponding vertex depends only upon the photon momentum: $\Gamma_\mu^{on}(p, p') \equiv \Gamma_\mu(p-p')$.

The crossing properties of the total current eq.(6) are considered in Appendix C.

C. Gauge dependence of the definition of off-shell EM FF

Since the one-body off-shell current is not conserved (see above), the definition of the off-shell FF is not gauge invariant in spite of the fact that the half-off-shell $\gamma^\star NN$ vertex satisfies the WTI for the three-point Green function. Indeed, if one of the nucleons is off-mass-shell, the vertex $\gamma^\star NN$ is only a part of the one-body reducible graph (nucleon-pole diagram). The corresponding one-body off-shell current cannot satisfy the next order WTI (for the Green function of the considered process, for example, the four-point one) itself, without a contribution of the "many-body" (contact) current, i.e. the regular piece of the amplitude. Therefore, the off-shell $\gamma^\star NN$ vertex cannot be considered independent of the full conserved current and the definition of the off-shell FF cannot be gauge invariant.

In practice the contribution of the off-shell Born current $J^{B-off}_{\mu\nu}$, which contains the off-shell EM FF, to the amplitude and any observables is proportional to the contraction:

$$T^g(P,q,q') \quad \sim \quad j^{lept}_\alpha(q)\ D^{(g)}_{\alpha\mu}(q^2)\ J^{B-off}_{\mu\nu}(P,q,q')\epsilon_\nu(q'), \tag{7}$$

where $D^{(g)}_{\alpha\mu}(q^2)$ is the propagator of the virtual photon and index g denotes its gauge; $\epsilon_\nu(q')$ is the polarization vector of the real photon, and $j^{lept}_\alpha(q)$ is a leptonic current which corresponds to the $e^-\gamma^\star e^-$ ($\gamma^\star e^+e^-$) vertex for space-like (time-like) photon, and which is conserved automatically (one-photon approximation)

$$q_\alpha\ j^{lept}_\alpha(q) \ = \ 0, \tag{8}$$

since the initial/final leptons are on-mass shell.

Let us consider the sensitivity of the amplitude $T^g(P,q,q')$, eq.(7), to the choice of the gauge for the virtual photon propagator. In general, any known gauge can be used for $D^{(g)}_{\alpha\mu}(q^2)$, but here we will consider only few the most common of them, namely the covariant Feynman ($\eta = 1$) and Landau ($\eta = 0$) gauge

$$D^{(g)}_{\alpha\mu}(q^2) = \frac{1}{q^2+i0}\{g_{\alpha\mu} + (1-\eta)\frac{q_\alpha q_\mu}{q^2}\}, \tag{9}$$

and "axial gauge"

$$D^{(g)}_{\alpha\mu}(q^2) = \frac{1}{q^2+i0}\{g_{\alpha\mu} + \frac{q_\alpha q_\mu}{(nq)^2}n^2 - \frac{n_\alpha q_\mu + q_\alpha n_\mu}{(nq)}\}, \tag{10}$$

where $n^2 = \pm 1, 0$ corresponds to "axial" ($A_3 = 0$), Weyl ($A_0 = 0$) and "light-cone" ($A_0 - A_3 = 0$) gauges. Substituting eq.(9) into eq.(7) and taking into account eq.(8), we see that the contribution of the $J^{B-off}_{\mu\nu}$ to the amplitude $T^g(P,q,q')$ has a *fixed value* which is the same for both covariant gauges (Feynman and Landau), since the leptonic current is conserved

$$T^{\eta=0}(P,q,q') = T^{\eta=1}(P,q,q').$$

However, in general the conservation of only the leptonic current in eq.(7) is not enough for a gauge invariant definition of the off-shell EM FF. Indeed, substituting eq.(10) into eq.(7) and taking into account that $n_\alpha j^{lept}_\alpha \neq 0$, we see that for any "axial" gauge the amplitude $T^g(P,q,q')$ differs from the "covariant" ones. Moreover, various "axial" gauges, which are *equivalent* in principle, will also lead to different results

$$T^{n^2=1}(P,q,q') \;\neq\; T^{n^2=-1}(P,q,q') \;\neq\; T^{n^2=0}(P,q,q').$$

Therefore, the definition of the off-shell EM FF in the one-body current, even through $\gamma^\star NN$ vertex which satisfy the WTI for the three-point EM Green function, is gauge dependent and such a form factors cannot be investigated in practice. Only exactly conserved hadron currents, like $J^N_{\mu\nu}$ from eq.(6), may lead to *physical* contributions to *observables*.

III. GAUGE INVARIANT DECOMPOSITION OF THE TOTAL NUCLEON CURRENT

The reducible half-off-shell $\gamma^\star NN$- vertex, satisfying the WTI (3), contains two terms[1] $\Gamma^\pm_\mu(q^2, p^2 \neq M^2)$ corresponding to the positive and negative energy states of the virtual nucleon

$$\Gamma_\mu = \Gamma^+_\mu(q^2,p^2)\,\frac{\not p + M}{2M} + \Gamma^-_\mu(q^2,p^2)\,\frac{-\not p + M}{2M}, \tag{11}$$

which can be expressed[1−3] through the *off-shell* EM FF $F^\pm_{1,2}(q^2,p^2)$:

$$\Gamma^{\pm}_{\mu}(q^2,p^2) = e\{F^{\pm}_1(q^2,p^2)\gamma_\mu + \frac{1-F^{\pm}_1(q^2,p^2)}{q^2}\not{q}q_\mu - \frac{\sigma_{\mu\nu}q_\nu}{2M}F^{\pm}_2(q^2,p^2)\}, \tag{12}$$

where $\sigma_{\mu\nu} = (\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$, and $\not{q} = \gamma_\mu q_\mu$.

For an on-mass-shell particle only $\Gamma^{+}_{\mu}(q^2, p^2 = M^2)$ exists, and $F^{+}_{1,2}(q^2,p^2)$ may be considered as the *physical* FF of the nucleon, contrary to $F^{-}_{1,2}(q^2,p^2)$.

For simplicity we will not consider the Pauli FF $F^{\pm}_2$ in eq.(12) explicitly. However, we stress that the procedure, similar to that one described below, may be directly apply to the full vertex in eq.(12).

At the photon point gauge invariance demands[2,10] $F^{\pm}_1(q^2 = 0, p^2) \equiv 1$. Therefore, assuming a regular behaviour of the *physical* FF as a function of q^2 and p^2 (which is evident below the pion threshold), we get a model independent representation

$$F^{+}_1(q^2,p^2) = F^{+}_1(q^2,M^2) + \frac{q^2}{M^2}\frac{p^2-M^2}{M^2}A^{+}(q^2,p^2). \tag{13}$$

Here the analytical function $A^{+}(q^2,p^2)$ contains all off-shell effects

$$A^{+}(q^2,p^2) = \sum_{i,k=1}^{\infty} a^{+}_{ik}\,(\frac{p^2-M^2}{M^2})^{i-1}(\frac{q^2}{M^2})^{k-1}, \tag{14}$$

and a^{+}_{ik} are constants, corresponding to the i,k- order derivatives at $p^2 = M^2$, $q^2 = 0$.

To derive a relation between off- and *on-shell* Born currents, we substitute eqs.(11,12) into eq.(4). Using eq.(13) and taking into account[6,9] that the numerator $p^2 - M^2$ cancels the denominators of the propagators in eq.(4), as well as the identities: $(\not{p} + M)S_0(p) = 1 + 2MS_0(p)$ and $(-\not{p} + M)S_0(p) = -1$, we express $J^{B-off}_{\mu\nu}$ from eq.(4) in terms of the gauge invariant *on-shell* Born current $J^{B-on}_{\mu\nu}$ (containing only *pole-type* singularities), and a regular term (containing only *one-body-irreducible* contributions); the latter consists of a gauge invariant part, $J^{GI}_{\mu\nu}$, and a non-gauge invariant one, $J^{non-GI}_{\mu\nu}$

$$J^{B-off}_{\mu\nu} = J^{B-on}_{\mu\nu} + J^{GI}_{\mu\nu} + J^{non-GI}_{\mu\nu}. \tag{15}$$

The explicit form of the various currents in eq.(15) is

$$J^{B-on}_{\mu\nu} = e^2\{\Gamma^{on}_{\mu}S_0(p'+q')\gamma_\nu + \gamma_\nu S_0(p-q')\Gamma^{on}_{\mu}\}. \tag{16}$$

$$J^{GI}_{\mu\nu} = \frac{e^2 q^2}{M^4}\{A^+(q^2, W^2)\tilde{\gamma}_\mu \not{q}' \gamma_\nu - \gamma_\nu \not{q}' \tilde{\gamma}_\mu A^+(q^2, W'^2)\}. \tag{17}$$

$$J^{non-GI}_{\mu\nu} = \frac{2e^2 q^2}{M^4}\{p'_\nu A^+(q^2, W^2) + p_\nu A^+(q^2, W'^2)\}\tilde{\gamma}_\mu + \frac{e^2}{2M}\{\Delta(W^2)\tilde{\gamma}_\mu \gamma_\nu + \gamma_\nu \tilde{\gamma}_\mu \Delta(W'^2)\}. \tag{18}$$

Here $\Delta(W^2) = F_1^+(q^2, W^2) - F_1^-(q^2, W^2)$, $W^2 = (p' + q')^2$, $W'^2 = (p - q')^2$, and

$$\Gamma^{on}_\mu = \Gamma^+_\mu(q^2, M^2) \quad ; \qquad \tilde{\gamma}_\mu = \gamma_\mu - \frac{\not{q} q_\mu}{q^2} \tag{19}$$

From eq.(15) we see that $off\text{-}shell\ effects$ in the $\gamma^\star NN$- vertex may be completely moved (see also [6-9]) to the regular piece of the amplitude (17,18), containing only irreducible graphs. This statement is general and does not depend upon the type of the EM process (it may be applied directly to the photo/electro- disintegration of a bound system).

Using eqs.(16-18), it is easy to see that every current in eq.(15) has a non-singular limit at $q^2 \to 0$, and in the photon point (real Compton scattering off a Dirac or scalar particle) off-shell effects vanish (as a consequence of gauge invariance)

$$J^{B-off}_{\mu\nu}(q^2 = 0) \equiv J^{B-on}_{\mu\nu}(q^2 = 0)$$

$$J^{GI}_{\mu\nu}(q^2 = 0) \ = \ J^{non-GI}_{\mu\nu}(q^2 = 0) \ = \ 0. \tag{20}$$

To calculate the contribution of the "minimal" contact current we substitute eq.(11) into eq.(5). Then, taking into account requirements of covariance, the integration in eq.(5) can be performed analytically, independent of the explicit form of the vertex function

$$-\frac{1}{e}J^C_{\mu\nu} = [\Gamma^+_\mu(W^2) - \Gamma^-_\mu(W^2)]\gamma_\nu + \gamma_\nu[\Gamma^+_\mu(W'^2) - \Gamma^-_\mu(W'^2)]$$
$$+ \frac{p'_\nu}{p'q}[\Gamma^+_\mu(W^2) - \Gamma^+_\mu(M^2)] - \frac{p_\nu}{pq}[\Gamma^+_\mu(W'^2) - \Gamma^+_\mu(M^2)]. \tag{21}$$

Substituting eqs.(12,13) into eq.(21), we find that the contact current (21) exactly coincides with the non-gauge invariant piece of the regular term (18), but with opposite sign

$$J^C_{\mu\nu} = -J^{non-GI}_{\mu\nu}. \tag{22}$$

Summing eqs.(22) and (15), we see that the "minimal" contact current (22) kills all non-gauge invariant terms in $J^{B-off}_{\mu\nu}$, and as a result, off-shell effects are completely cancelled in the "longitudinal" part (in the charge operators) of the total conserved nucleon current $J^N_{\mu\nu}$

$$J^N_{\mu\nu} = J^{B-off}_{\mu\nu} + J^C_{\mu\nu} = J^{B-on}_{\mu\nu} + J^{GI}_{\mu\nu}, \tag{23}$$

where $q_\mu J^{B-on}_{\mu\nu} = J^{B-on}_{\mu\nu} q'_\nu = q_\mu J^{GI}_{\mu\nu} = J^{GI}_{\mu\nu} q'_\nu = 0$, and only the *new* "transverse" contact current $J^{GI}_{\mu\nu} \sim \tilde{\gamma}_\mu \sigma_{\nu\alpha} q'_\alpha$, caused by the Dirac magnetic moment, depends on off-shell effects. In case of VCS off a scalar particle the cancellation of off-shell effects is complete. As it may be seen from eqs.(17,18), the expansion (14) was not used to derive eq.(23).

Therefore, we obtained a novel decomposition (23) of the total conserved nucleon current into two gauge invariant pieces $J^{B-on}_{\mu\nu}$ and $J^{GI}_{\mu\nu}$ which shows: 1) the introduction of off-*shell* FF in the "reducible" $\gamma^\star NN$ vertex is not gauge invariant and depends on the representation of the total conserved current; 2) off-shell effects may be distributed over the $\gamma^\star NN$- vertex and the contact current in different ways; 3) off-shell effects from one-body currents are completely cancelled in the longitudinal components of the total conserved current by the "minimal" contact current; 4) F_1^- is removed from the total conserved nucleon current and does not influence the observables. Therefore, a representation of the total conserved current such that the off-shell effects enter only in the *non-pole* gauge invariant piece and only through Dirac magnetic operators exists for both space- and time-like regions. This enables one to introduce an "on-shell extrapolated" FF for the "unphysical" region in a gauge invariant way.

Note, that inclusion of the $F_2^\pm$ form factors in the same manner does not change the structure and the physical meaning of eq.(23). This leads just to only an additional term in $J^{GI}_{\mu\nu}$, containing the physical FF F_2^+ only, proportional to:

$$\left(\frac{d}{dp^2} F_2^+(q^2, p^2)\right)_{p^2=M^2} \{\sigma_{\mu\alpha}\sigma_{\beta\nu} + \sigma_{\beta\nu}\sigma_{\mu\alpha}\} q_\alpha q'_\beta.$$

There are also two additional terms in $J^{non-GI}_{\mu\nu}$, including both $F_2^\pm$ form factors; we do not present these terms here, since they are cancelled by the corresponding terms from the

"minimal" contact current, in the same manner as was shown above for $F_1^{\pm}$. Therefore, we can conclude, that the form factor F_2^- is also removed from the total conserved current, similary to F_1^-, and does not influence the observables.

We stress that the real Compton scattering amplitude off a Dirac (or scalar) particle cannot contain any off-shell effects at all, as a consequence of gauge invariance.

Finally, all results and conclusions, obtained above, are valid for both space/time-like regions, and the definition of the *on-shell Born* current is *unique* under the condition that corresponding $\gamma^\star NN$ vertices satisfy the WTI on the *operator level* (this is always true for a *Dirac* proton).

IV. CONSERVED CURRENT IN TERMS OF IRREDUCIBLE VERTICES

Here we will briefly repeat the same procedure of the "minimal substitution" of the EM field in the *left-hand* side of (1), which contains full propagators and an irreducible vertex. In this case minimal insertion of the second (real) photon into external lines (full renormalized nucleon propagators) generates[10], similar to eq.(4), an alternative expression for the one-body *off-shell* Born current

$$\tilde{J}^{B-off}_{\mu\nu} = \Gamma^{\rm ir}_\mu(p,p'+q')S(p'+q')\Gamma^{ir}_\nu(p'+q',p') + \Gamma^{ir}_\nu(p,p-q')S(p-q')\Gamma^{\rm ir}_\mu(p-q',p'). \quad (24)$$

Note, eq.(24) is formally the same as the one used in ref [9] for the contribution of the "class A" diagrams. However, in this paper the *irreducible* γNN vertex (with a real photon) is now the "minimal" three-point EM Green's function connected with inverse propagator[10] (see also [12])

$$\left\{ \begin{array}{c} \Gamma^{ir}_\nu(p'+q',p') \\ \Gamma^{\rm ir}_\nu(p,p-q') \end{array} \right\} = \pm e \int_0^1 \frac{d\lambda}{\lambda}\frac{d}{dq'_\nu} \left\{ \begin{array}{c} S^{-1}(p'+\lambda q') \\ S^{-1}(p-\lambda q') \end{array} \right\}, \quad (25)$$

and, as a result, satisfies the WTI (2) automatically. Eq.(25) is a consequence[10] of the "minimal coupling" of the second (real) photon to the full renormalized propagator, and follows from the eqs.(21)-(22) in the first ref. [10]. The *irreducible* $\gamma^\star NN$ vertices (with

virtual photons), $\Gamma^{\rm ir}_{\mu}(p'+q',p)$ and $\Gamma^{\rm ir}_{\mu}(p-q',p')$, satisfy the WTI (2) also[13]. However, for these we do not need a "minimal" procedure, and their Lorentz-spin structure may be described by the same equations like eqs.(11, 12).

The minimal photon insertion inside the *irreducible* $\gamma^{\star}NN$ vertex gives rise the new contact current which has the same structure as eq.(5); however it contains *irreducible* vertices instead of *reducible* ones

$$\tilde{J}^{C}_{\mu\nu} = -e\int_0^1 \frac{d\lambda}{\lambda}\frac{d}{dq'_{\nu}}\{\Gamma^{\rm ir}_{\mu}(p,p'+\lambda q') - \Gamma^{\rm ir}_{\mu}(p-\lambda q',p')\}. \tag{26}$$

Using the WTI (2) and eqs.(25), as well as taking into account that $\bar{u}(p)S^{-1}(p) = \bar{u}(p)S_0^{-1}(p) = S^{-1}(p')u(p') = S_0^{-1}(p')u(p') = 0$, it is easy to get convinced that the derivatives of $\tilde{J}^{C}_{\mu\nu}$ cancel corresponding derivatives of $\tilde{J}^{B-off}_{\mu\nu}$, independent of the explicit form of the vertices, and the *new* total nucleon current

$$\tilde{J}^{N}_{\mu\nu} \quad = \quad \tilde{J}^{B-off}_{\mu\nu} \quad + \quad \tilde{J}^{C}_{\mu\nu}\ , \tag{27}$$

defined in the terms of the *irreducible* vertices and $full$ renormalized propagators, is exactly conserved

$$q_{\mu}\tilde{J}^{N}_{\mu\nu} \quad = \quad \tilde{J}^{N}_{\mu\nu}q'_{\nu} \quad = \quad 0.$$

To establish a relation between two gauge invariant nucleon currents, $J^{N}_{\mu\nu}$ from (6) and $\tilde{J}^{N}_{\mu\nu}$ from (27), we have to take into account the connection between "full" renormalized and "free" Feynman propagators

$$S^{-1}(p) = \not{p} + M - \Sigma(p) \quad ; \quad S_0^{-1} = \not{p} + M \quad ; \quad S_0^{-1}(p) - S^{-1}(p) \ = \ \Sigma(p), \tag{28}$$

where the mass-operator $\Sigma(p)$ satisfies the following conditions:

$$\Sigma(p)|_{p^2=M^2} = 0 \quad ; \quad \frac{d}{d\not{p}}\Sigma(p)|_{p^2=M^2} = 0. \tag{29}$$

Substitution of eq.(28) into eq.(25) gives the connection between the "minimal" γNN *irreducible* vertex and the *reducible* one

$$\Gamma^{\rm ir}_{\nu}(p'+q',p') = e\gamma_{\nu} - e\int_0^1 \frac{d\lambda}{\lambda}\frac{d}{dq'_{\nu}}\Sigma(p'+\lambda q'), \tag{30}$$

Using eq.(30) and taking into account that $\Gamma^{\rm ir}_\mu(p,p'+q')S(p'+q') = \Gamma_\mu(p,p'+q')S_0(p,p'+q')$ and $S(p-q')\Gamma^{\rm ir}_\mu(p-q',p') = S_0(p-q')\Gamma_\mu(p-q',p')$, we can identify in eq.(24) the piece that contain only *reducible* vertices and *free* Feynman propagators, i.e. the part which exactly coincides with $J^{B-off}_{\mu\nu}$ from (4), and a correction term comming from the mass-operator (see Appendix D):

$$\tilde{J}^{B-off}_{\mu\nu} \quad = \quad J^{B-off}_{\mu\nu} \quad + \quad \delta J^{B-off}_{\mu\nu}, \tag{31}$$

The same procedure for the contact current $\tilde{J}^{C}_{\mu\nu}$ (26) (see Appendix D) allows us to identify the piece which contains only *reducible* vertices and, of course, exactly coincides with our old contact current $J^{C}_{\mu\nu}$ from (5) plus correction term $\delta J^{C}_{\mu\nu}$ caused also by the mass-operator:

$$\tilde{J}^{C}_{\mu\nu} \quad = \quad J^{C}_{\mu\nu} \quad + \quad \delta J^{C}_{\mu\nu}, \tag{32}$$

The direct calculation of $\delta J^{C}_{\mu\nu}$ gives (see Appendix D):

$$\delta J^{C}_{\mu\nu} \quad = \quad -\delta J^{B-off}_{\mu\nu}. \tag{33}$$

Thus, substituting eqs.(31,32) into eq.(27) and taking into account eq.(33) we conclude

$$\tilde{J}^{N}_{\mu\nu} \quad \equiv \quad J^{N}_{\mu\nu}. \tag{34}$$

Therefore, in the framework of the "minimal" scheme for the gauge field the corrections, connected with the mass-operator, to the off-shell Born current and to the contact current cancel one another in the total conserved current. As a result, its definition in terms of the ir-reducible vertices and full renormalized propagator as well as in terms of the reducible vertices and free Feynman propagators is *equivalent*.

Finally, eq.(34) is also satisfied if both photons are real (this can be shown by using the above described procedure with additional symmetrization[14] in eq.(26) on photon lines). Note, the identity (34) leads to some restrictions for the off-shell effects at the "photon-point", as a consequence of the gauge invariance. Indeed, in accordance with eq.(20), the right hand side of eq.(34), i.e. current $J^{N}_{\mu\nu}$, does not contain any off-shellness for the real

Compton scattering on a Dirac (or scalar) particle. This means that the off-shell effects in the left hand side of eq.(34), i.e. in the current $\tilde{J}^N_{\mu\nu}$, caused by the self-energy part, appear in the correction terms only, and cancel in the total gauge invariant amplitude in a model independent way.

V. ILLUSTRATION: TIME-LIKE VCS IN "UNPHYSICAL" REGION

As an illustration we consider the possibility to introduce an on-shell extrapolated Dirac FF in "unphysical" region and to investigate it experimentally in the di-lepton photoproduction off a proton. All information about time-like FF is concentrated in the VCS amplitude which is much smaller than Bethe-Heitler (BH) one in general. Therefore, their separation by measuring the C- odd e^+e^-- asymmetry in the $\gamma p \to p e^+ e^-$ reaction has been proposed[14]. The selection of "longitudinal" photons $\mathbf{q}^2 \ll q^2 \ll 4M^2$ in the collinear kinematics provides[14] a strong suppression of the Pauli FF F_2 and the contribution from resonances, which are excited mainly by magnetic (transverse) transitions. As a result, for these "longitudinal" photons the *physical* proton resembles a Dirac one[14]. Taking into account that squares of BH/VCS amplitudes are C- even under permutation of e^+e^- ($T^{BH,VCS}_{-+} = T^{BH,VCS}_{+-}$), while their interference is C- odd ($T^{int}_{-+} = -T^{int}_{+-}$), one can introduce the e^+e^-- asymmetry[14] ($K^{-1}_{-+} = E_- + E_+ \cos(\theta) + p_0$):

$$A_{e^+e^-} \;=\; \frac{K^{-1}_{+-}\sigma_{+-} - K^{-1}_{-+}\sigma_{-+}}{K^{-1}_{+-}\sigma_{+-} + K^{-1}_{-+}\sigma_{-+}} \;\equiv\; \frac{T^{int}_{-+}}{T^{BH}_{-+} + T^{VCS}_{-+}} \;\approx\; T^{int}_{-+} \,/\, T^{BH}_{-+}\,,$$

where $E_\pm$ and θ are the energy of positron/electron and the angle between them. We use c.m. system with z- axis fixed along 3-momentum of the virtual photon $\mathbf{q}$, and $\sigma_{+-} = d^5\sigma_{+-}/dE_- d\Omega_- d\Omega_+$.

The $A_{e^+e^-}$- asymmetry is shown in fig.1 as a function of q^2 at $E_\gamma = 0.65$ GeV, $\theta = 165^o$, $\theta_{\gamma\gamma^\star} = 0^o$ (collinear kinematics with almost "longitudinal" photons: $\mathbf{q}^2 \ll q^2$). To get an idea about the sensitivity of asymmetry to the q^2- dependence, we compare several models for $F_1(q^2)$ assuming the vector meson dominance (VMD). The chiral quark-bag model[15],

containing the coupling of the photon to the meson cloud (VMD piece) and directly to the quark core, dispersion relation calculations[16], including constraints from the hadron-channel unitarity (part corresponding to VMD) and perturbative QCD, as well as the well known "dipole fit" were extrapolated to the time-like "unphysical" region. All of them practically coincide and fit experimental data well in the space-like region. When extended into the time-like sector, they give rise to a resonance-like peak corresponding to the VMD contribution[4], and different results even at small positive q^2. We have also calculated the asymmetry for the "point-like" proton, when $F_1(q^2) = 1$.

It is interesting to observe that due to the $\pi\pi$- continuum contribution to the isovector spectral function[16], dispersion relation calculations (VMD + PQCD) predict practically the same asymmetry as a pure meson cloud piece of the chiral quark-bag model, while the full quark-bag model gives the result which is very close to the dipole fit prediction. The comparison of these models with the "point-like" proton prediction shows a strong sensitivity of the asymmetry to the EM structure of the proton. All curves in fig.1 (except the dotted curve) were obtained without off-shell effects: $F_1^+(q^2, W^2) \rightarrow F_1^+(q^2, M^2)$, and expansion (14) was not used at all.

To estimate the size of off-shell effects (which come only from $J_{\mu\nu}^{GI}$ in a new representation (17) of the full conserved current) we took into account the leading order in the expansion (13) on the hadron virtuality: $a_{11} \neq 0$ and $a_{ik} = 0$, if $i, k > 1$. A small difference between two lowest curves in the picture, dotted and double-dashed ones, obtained for the "point-like" proton, but with $a_{11} = 1$ and 0, respectively, reflects the insignificant role of off-shell effects in the full conserved current (6). Therefore, selection of "longitudinal" photons[14] in collinear kinematics of $\gamma p \rightarrow p e^+ e^-$ reaction can provide a practically model-independent investigation of the "on-shell extrapolated" Dirac FF in the "unphysical" region.

VI. DISCUSSION AND CONCLUSIONS

In this paper "off-shell Born currents" (irregular pieces of the amplitude) and "contact currents" (regular pieces of the amplitude) were obtained "simul;taniously" from the same principle in an explicit form, using only gauge properties of the EM field, in terms of the well-known structure[1−4] of the half-off-shell vertex $\gamma^\star NN$. So, a completely consistent treatment of irregular and regular pieces of the total VCS amplitude off a Dirac proton was achieved without assuming a dynamical model for off-*shellness*. This allows us to consider any re-distributions and cancellations of the off-shell effects inside the total gauge invariant current in a model independent way for the first time. Two different representations of the "minimal" conserved current in terms of the *reducible* vertices and $free$ Feynman propagators, as well as *ir-reducible* vertices and $full$ renormalized propagators were considered and their equivalence was shown.

To summarize, the definition of the off-shell EM FF through reducible or irreducible $\gamma^\star NN$ vertices is not gauge invariant even if these vertices satisfy WTI for the 3-point EM Green function. For the pure EM processes, like VCS, off-shell effects in the *reducible* $\gamma^\star NN$ vertices may not only be moved to the regular part of the amplitude (analogously to the result[8.9] for *ir-reducible* vertices), but, and it is more important, they are **cancelled** in the longitudinal components of the total conserved current by the "minimal" contact current, when pole and contact amplitudes are defined consistently. Therefore, "charge operators" are completely free from the off-shellnes due to gauge constraint. This means that indeed such a model independent representation of the total conserved current, when off-shell effects enter only in the transverse (magnetic) gauge invariant *non-pole* part, exists for both space/time-like regions. This enables one to introduce an "on-shell extrapolated" FF for the "unphysical" region in a gauge invariant way which could be investigated in practice. The *unphysical* FF $F^-_{1,2}$ is removed from the full amplitude due to current conservation and does not influence the observables.

The off-shellness in the "irreducible" vertices, which appears even at photon point and

which is caused by the mass-operator corrections, is cancelled in the "minimal" conserved current in a model independent way, since self-energy corrections to the Born and contact currents are the same, but they have opposite signs. In general this means that the requirements of gauge invariance impose model independent restrictions on the off-shell properties of the charge operators at the "photon point". As a result, any off-shell effects cannot exist in the real Compton scattering amplitude off a scalar (or Dirac) particle as a consequence of a gauge invariance only. Indeed, in this case any off-shellness which appear in the "irreducible representation" may be canncelled simply by the transition to the theory in terms of the *reducible* vertices and $free$ Feynman propagators.

Let us stress that all above mentioned results were obtained in the framework of the "minimal substitution" scheme, on the basis of only fundamental requirements, such as covariance, CPT and gauge invariance, without any model suppositions about off-shell behaviour of the nucleon form factors. Of course, such a model independent consideration is possible only for a Dirac or scalar particles when a consistent treatment of the regular and irregular parts of the total amplitude may be done using only gauge nature of the EM field. Indeed, "pure transverse" pieces of the Born and contact currents (associated with *real photon*), including "abnormal" (Pauli) part of the γNN vertex in the one-body current, cannot be obtained in the "minimal" scheme only on the basis of the gauge constraint for 3- and 4-point EM Green functions (with gauge field). Additional gauge constraint for 4- and 5-point EM Green functions should be considered on the level of at least one-loop diagrams[10] corresponding to "vertex corrections" to the "minimal" truncated γNN Green functions. This evidently requires some knowledge of the strong interaction structure, i.e. strong dynamics in the pure hadronic 3- and 4-point Green functions (without gauge field). However, the off-shellness arising from the "magnetic" couplings in the $\gamma^{\star} NN$ vertex (associated with *virtual photon*, see eq.(12)) may also be moved to the contact current in a manner similar to eqs. (13), (15)-(18).

Finally, measurements of $A_{e^+e^-}$- asymmetry in the $\gamma p \to pe^+e^-$ reaction with "longitudinal" photons, with a mass much more larger than 3-momentum, can be used to investi-

gate the on-shell extrapolated Dirac FF $F_1^+(q^2, M^2)$ of the proton in the time-like region $4m_e^2 < q^2 < 4M^2$.

When this work was completed, some attempt to estimate the role of off-shell effects in the VCS was published[17].

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VII. APPENDIX A

The general form of the Ward-Takahashi identity for the *reducible* three-point Green function with the EM field (vertex γNN in our case) is well known

$$q_\mu \Gamma_\mu(p, p') = e S_0^{-1}(p)[S(p') - S(p)] S_0^{-1}(p'). \qquad (A.1)$$

Using the asymptotic relation $\lim_{p^2 \to M^2} S_0^{-1}(p) S(p) = 1$ the gauge constraint for the $half$-off-$shell$ reducible vertex becomes (let us put p is an on-shell momentum, while p' is an off-shell one: $p^2 = M^2$, $p'^2 \neq M^2$, and $u(p)$ is a Dirac bi-spinor, i.e. a wave function of the on-shell particle):

$$\bar{u}(p) q_\mu \Gamma_\mu(p, p') = e \bar{u}(p) S_0^{-1}(p)[S(p') - S(p)] S_0^{-1}(p') = e \bar{u}(p) \{S_0^{-1}(p) - S_0^{-1}(p')\} \qquad (A.2)$$

Therefore from eqs.(A.1) and (A.2) we see that the simple form of eq.(3) for the Ward-Takahashi identity may be used for the half-off-shell γNN vertex.

VIII. APPENDIX B

The structure of the VCS off-$shell$ $Born$ current in eq.(4) will be considered here on the basis of the "minimal insertion" of the EM field. We start with the definition of the reducible $\gamma^\star NN$ vertex $\Gamma_\mu(p', p)$ in terms of the irreducible $\Gamma_\mu^{\rm ir}(p', p)$ one, eq.(1).

Since the *reducible* vertex in eq.(1) is surrounded by free propagators, we have to deal with the "minimal insertion" of the external EM field $A_\nu(q')$, corresponding to the real (second) photon, into the "free" external hadron lines in the right-hand side of eq.(1)

$$S_0(p')\Gamma_\mu(p',p)S_0(p) \quad \rightarrow \quad S_0(p'-eA)\Gamma_\mu(p',p)S_0(p-eA) \equiv G_\mu(p',p,\{A\}) \qquad (B.1)$$

Then, the corresponding EM current is:

$$J^{B-off}_{\mu\nu} = S_0^{-1}(p') \;\; [\frac{\delta}{\delta A_\nu} G_\mu(p',p,\{A\})]_{A\to 0} \;\; S_0^{-1}(p), \qquad (B.2)$$

Using (B.1) and (B.2) we obtain the off-shell Born current for VCS in the form of eq.(4).

The same result may be obtained if one takes into account that the "minimal" three-point Green function, corresponding to the "minimal" coupling of the real photon to the external hadron lines, is connected with the corresponding inverse propagator[10] (in our case, see eq.(1), it is the $free$ propagator):

$$\Gamma_\nu(p-q',p) = -e\int_0^1 \frac{d\lambda}{\lambda}\frac{d}{dq_\nu} S_0^{-1}(p-\lambda q') = e\gamma_\nu,$$

IX. APPENDIX C

A general Lorentz covariant, crossing symmetry, CPT and gauge invariant expression of the off-*shell* nucleon current for the space/time-like VCS, containing independently crossing symmetry off-shell Born term and contact current both in an explicite form as a function of only half-off-shell nucleon form factors in the $\gamma^\star NN$ vertex, was obtained recently[14].

In the particular case, when one photon is *real* and the other *virtual*, the crossing transformation connects the amplitudes of the *time-like* and the *space-like* VCS. Indeed from eqs.(4,5) a total gauge invariant current for time-like VCS ($q^2 > 0$, $q'^2 = 0$) is:

$$e^{-1}J^N_{\mu\nu}(\gamma N \rightarrow N\gamma^\star) = \Gamma_\mu(p,p'+q')S_0(p'+q')\gamma_\nu + \gamma_\nu S_0(p-q')\Gamma_\mu(p-q',p')$$
$$- \int_0^1 \frac{d\lambda}{\lambda}\frac{d}{dq'_\nu}\{\Gamma_\mu(p,p'+\lambda q') - \Gamma_\mu(p-\lambda q',p')\}. \qquad (C.1)$$

Equation (C.1) under crossing transformation $q \to -q'$, $q' \to -q$, $\mu \leftrightarrow \nu$ generates a gauge invariant current for space-like VCS ($q^2 = 0$, $q'^2 < 0$):

$$e^{-1} J^N_{\nu\mu}(\gamma^\star N \to N\gamma) = \Gamma_\nu(p, p'-q) S_0(p'-q)\gamma_\mu + \gamma_\mu S_0(p+q)\Gamma_\nu(p+q, p')$$
$$- \int_0^1 \frac{d\lambda}{\lambda} \frac{d}{dq'_\mu} \{\Gamma_\nu(p+\lambda q, p') - \Gamma_\nu(p, p'-\lambda q)\}, \qquad (C.2)$$

Although the currents (C.1) and (C.2) are not the same (since the initial and final photons are different), they are connected by crossing transformations.

At the photon point (both photons are real) due to WTI(3) all reducible vertices $\Gamma_\mu(p, p')$ in eqs.(15,5) become simply a constant ($e\gamma_\mu$). This means that for real photons the reducible half-off-shell γpp vertex of the Dirac proton coincides with the on-shell one and does not contain any off-$shellness$ (consequence of the gauge constraint[2,10]). As a result, $J^{B-off}_{\mu\nu}(q^2 = q'^2 = 0) = J^{B-on}_{\mu\nu}(q^2 = q'^2 = 0)$, $J^C_{\mu\nu}(q^2 = q'^2 = 0) = 0$, and in the case of two real photons current (23) is crossing symmetry

$$J^N_{\mu\nu}(P, q', q)|_{q^2=q'^2=0} = J^N_{\nu\mu}(P, -q, -q')|_{q^2=q'^2=0}$$

X. APPENDIX D

First, we derive the connection between two different off-shell Born currents $\tilde{J}^{B-off}_{\mu\nu}$ and $J^{B-off}_{\mu\nu}$, defined in terms of the *ir-reducible* vertices and $full$ renormalized propagators, and in terms of the *reducible* vertices and $free$ Feynman propagators, respectively. Starting from eq.(24), and accounting for the definition (1), we can make transition from the irreducible vertices of the virtual photons and full propagators to the reducible ones and free propagators, while for the real photons we have still irreducible vertices:

$$\tilde{J}^{B-off}_{\mu\nu} = \Gamma_\mu(p, p'+q') S_0(p'+q') \Gamma^{ir}_\nu(p'+q'.p') + \Gamma^{ir}_\nu(p, p-q') S_0(p-q') \Gamma_\mu(p'+q'.p').$$

The substitution of eq.(30) for irreducible γNN vertices allows us to decompose the "new" off-shell Born current (24) into two terms. The first one is the "old" Born current $J^{B-off}_{\mu\nu}$

from eq.(4), while the second one, $\delta J_{\mu\nu}^{B-off}$, is a correction term coming from the mass-operator:

$$\tilde{J}_{\mu\nu}^{B-off} = \Gamma_\mu(p,p'+q')S_0(p'+q')(e\gamma_\nu) + (e\gamma_\nu)S_0(p-q')\Gamma_\mu(p-q'.p') + \delta J_{\mu\nu}^{B-off}, \quad (D.1)$$

$$\delta J_{\mu\nu}^{B-off} = -e\int_0^1 \frac{d\lambda}{\lambda}\{\Gamma_\mu(p,p'+q')S_0(p'+q')[\frac{d}{dq'_\nu}\Sigma(p'+\lambda q')]-$$

$$-[\frac{d}{dq'_\nu}\Sigma(p-\lambda q')]S_0(p-q')\Gamma_\mu(p-q',p')\}. \quad (D.2)$$

From eqs.(D1,2), and using eq.(30), we immediately get the relation between two off-shell Born currents in the form of eq.(31).

Second, we derive the connection between two the different contact currents $J_{\mu\nu}^C$ and $\tilde{J}_{\mu\nu}^C$ which are defined through irreducible and reducible vertices, respectively. Starting from eq.(26), we can rewrite it as

$$\tilde{J}_{\mu\nu}^C = -e\int_0^1 \frac{d\lambda}{\lambda}\frac{d}{dq'_\nu}\{\Gamma_\mu^{\rm ir}(p,p'+\lambda q')S(p'+\lambda q')S^{-1}(p'+\lambda q')-$$

$$-S^{-1}(p-\lambda q')S(p-\lambda q')\Gamma_\mu^{\rm ir}(p-\lambda q',p')\}. \quad (D.3)$$

Using the definition (1), and the relation between $free$ and $full$ propagators (28), we can make the transition from the irreducible vertices to the reducible ones:

$$\tilde{J}_{\mu\nu}^C = -e\int_0^1 \frac{d\lambda}{\lambda}\frac{d}{dq'_\nu}\{\Gamma_\mu(p,p'+\lambda q')S_0(p'+\lambda q')[S_0^{-1}(p'+\lambda q') - \Sigma(p'+\lambda q')]-$$

$$-[S_0^{-1}(p-\lambda q') - \Sigma(p-\lambda q')S_0(p-\lambda q')\Gamma_\mu(p-\lambda q',p')\}. \quad (D.4)$$

Isolating in (D.4) terms containing only reducible vertices, we see that the "new" contact current (26) may be presented in the form (32), i.e. expressed through the "old" contact current $J_{\mu\nu}^C$, defined by eq.(5), and an additional correction term $\delta J_{\mu\nu}^C$:

$$\delta J_{\mu\nu}^C = e\int_0^1 \frac{d\lambda}{\lambda}\{[\frac{d}{dq'_\nu}\Gamma_\mu(p,p'+\lambda q')]S_0(p')\Sigma(p') + \Gamma_\mu(p,p'+q')[\frac{d}{dq'_\nu}S_0(p'+\lambda q')]\Sigma(p')+$$

$$+\Gamma_\mu(p,p'+q')S_0(p'+q')[\frac{d}{dq'_\nu}\Sigma(p'+\lambda q')] - [\frac{d}{dq'_\nu}\Sigma(p-\lambda q')]S_0(p-q')\Gamma_\mu(p-q',p')-$$

$$-\Sigma(p)[\frac{d}{dq'_\nu}S_0(p-\lambda q')]\Gamma_\mu(p-q',p') - \Sigma(p)S_0(p)[\frac{d}{dq'_\nu}\Gamma_\mu(p-q',p')]\}. \quad (D.5)$$

Then, taking into account (29), and omitting all terms which are proportional to $\Sigma(p)$ and $\Sigma(p')$ since $p^2 = p'^2 = M^2$, we can write down the "contact current correction":

$$\delta J^C_{\mu\nu} = e \int_0^1 \frac{d\lambda}{\lambda} \{ \Gamma_\mu(p, p' + q') S_0(p' + q') [\frac{d}{dq'_\nu} \Sigma(p' + \lambda q')] -$$

$$[\frac{d}{dq'_\nu} - \Sigma(p - \lambda q')] S_0(p - q') \Gamma_\mu(p - q', p') \}. \qquad (D.6)$$

Comparing eqs. (D.2) and (D.6), we immediately get eq.(33).

REFERENCES

[1] A.M. Bincer, Phys. Rev. **118**, 855 (1960).

[2] H.W.L. Naus and J.H. Koch, Phys. Rev. C**35**, 2459 (1987).

[3] P.C. Tiemeijer and J.A. Tjon, Phys. Rev. C**42**, 599 (1990).

[4] H.C. Dönges, M. Schäfer and U. Mosel, Phys. Rev. C**51**, 950 (1995).

[5] S. Kamefuchi, L. O'Raifeartaigh and A. Salam, Nucl. Phys. **28** , 529 (1961); S. Coleman, J. Wess and Bruno Zumino, Phys. Rev. **177**, 2239 (1969); H. Georgi, Nucl. Phys. **B 361**, 339 (1991).

[6] A. I. L. L'vov, Int. J. Mod. Phys. A **8**, 5267 (1993).

[7] S. Scherer and H.W. Fearing, Phys. Rev. C**51**, 359 (1995).

[8] P.A.M. Guichon, G.Q. Liu and A.W. Thomas, Nucl. Phys. **A591**, 606 (1995).

[9] S. Scherer, A.Yu. Korchin and J.H. Koch, Phys. Rev. C**54**, 904 (1996).

[10] S.I. Nagorny, Yu.A. Kasatkin, I.K. Kirichenko and E.V. Inopin, Sov. J. Nucl. Phys. **49**, 465 (1989); **53**, 228 (1991).

[11] M. Vanderhaegen, Phys. Lett. **B368**, 13 (1996).

[12] K. Ohta, Phys. Rev. C**40**, 1335 (1989).

[13] C. Itzykson and J.-B. Zuber. Quantum Field Theory. McGraw-Hill, New York, 1985

[14] A.E.L. Dieperink and S. Nagorny, Phys. Lett. **B 397**, 20 (1997).

[15] G.E. Brown, M. Rho and W. Weise, Nucl. Phys. **A454**, 669 (1986).

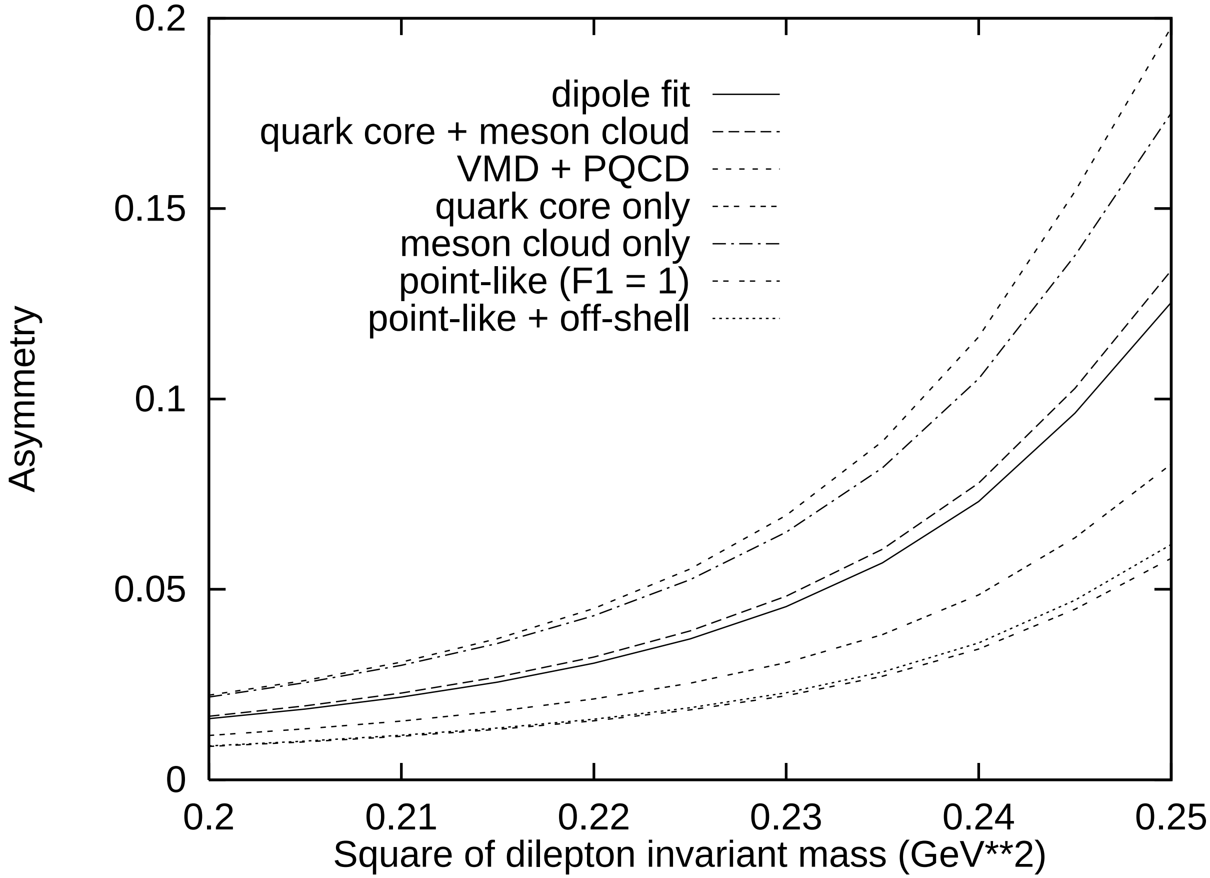
[16] P. Mergell, Ulf-G. Meissner and D. Drechsel, Nucl. Phys. **A596**, 367 (1996).

[17] A.Yu. Korchin, O. Scholton and F. de Jong, Phys. Lett. **B 402**, 1 (1997).

Figure Captions

fig.1

Predictions for the asymmetry $A_{e^+e^-}$ for different models for $F_1(q^2, M^2)$.


dipole fit
quark core + meson cloud
VMD + PQCD
quark core only
meson cloud only
point-like (F1 = 1)
point-like + off-shell
Asymmetry
0.2
0.15
0.1
0.05
0
0.2
0.21
0.22
0.23
0.24
0.25
Square of dilepton invariant mass (GeV**2)